The Information Dynamics of Synchrony

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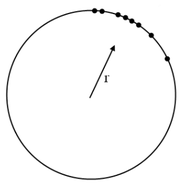
Synchrony

The Kuramoto Model

- Each node attempts to oscillate at its own native frequency ω_i, but is influenced by the difference of its own state θ_i to those of its neighbours θ_j.

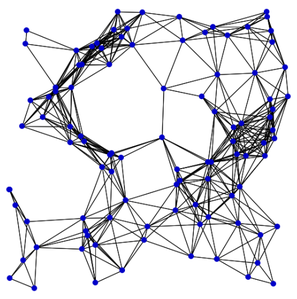
$$\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} K_{ij} \sin(\theta_j - \theta_i)$$

- The centroid of the phases of the population around the unit circle measures the order *r* of the system.



Scattered Network Topology

- Simulating the topology of wireless sensor networks, nodes are scattered on a surface and linked to all nodes within a fixed range.



Information Dynamics

Information Theory

- Method of quantising the amount of useful information in a message.
- Key measure is the Shannon Entropy, giving the amount of bits required to encode a message *x*, and thus its information content.

$$H_X = -\sum_{x} p(x) \log_2 p(x)$$

Active Information Storage

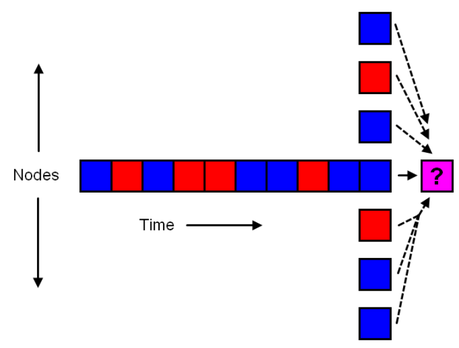
- How much information about the next state x_{t+1} is contained in the node's own past x^k

$$a_X(t+1) = \lim_{k\to\infty} i(x_t^k; x_{t+1})$$

Information Transfer

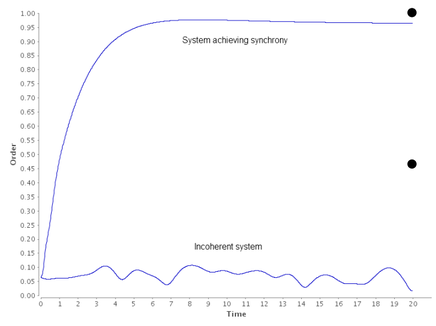
- How much the state of a single neighbor *Y* informs us about the next state of this node *X*, given the node's own past.

$$T_{Y\to X} = \lim_{k\to\infty} p(x_{t+1}, x_t^k, y_t) \log_2 \frac{p(x_{t+1}|x_t^k, y_t)}{p(x_{t+1}|x_t^k)}$$



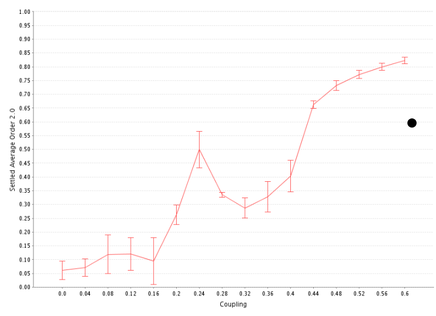
Behaviour of the Model

Progression of order over time



- Incoherent systems are unable to increase their order even to $t = \infty$
- Coherent systems are able to quickly organise themselves into a highly ordered state

Settled State Order vs Coupling



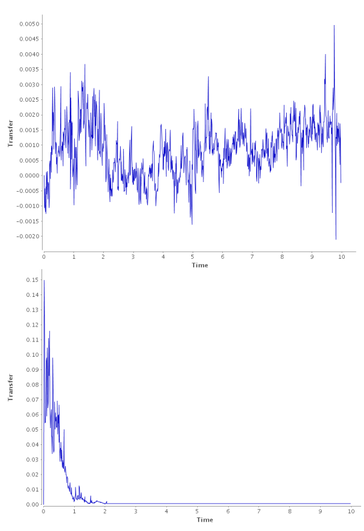
- Below critical coupling no nodes are able to synchronise together
- As *K* increases there is a phase transition, nodes start to group, pulling in more nodes until complete synchrony

Settled State Order vs Range

- As the connectivity of the network increases, the system as a whole is able to communicate more readily, lowering the critical coupling required for synchrony

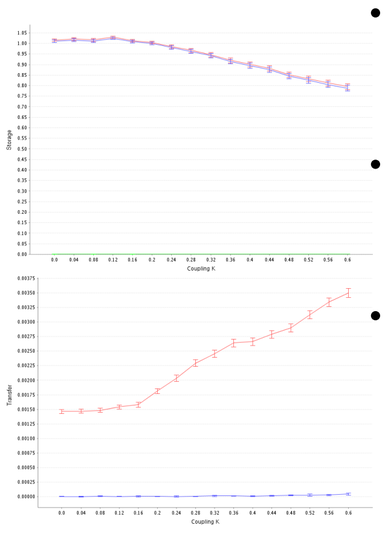
Results in Healthy Networks

System Mean Information Transfer over Time



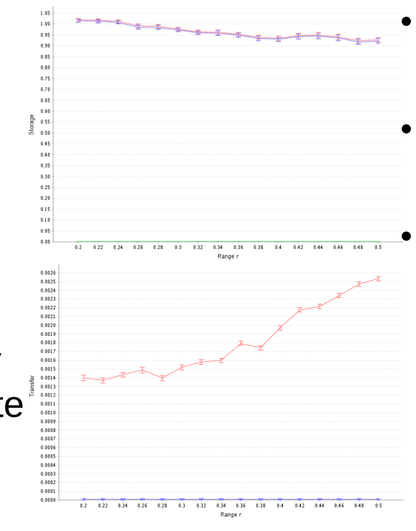
- Top: Stays at very low values in an incoherent system
- Bottom: A system able to synchronise has extremely high values in the transient stage, dropping to almost zero much faster than the when order parameter shows complete synchrony is achieved

System Mean Dynamics vs Coupling



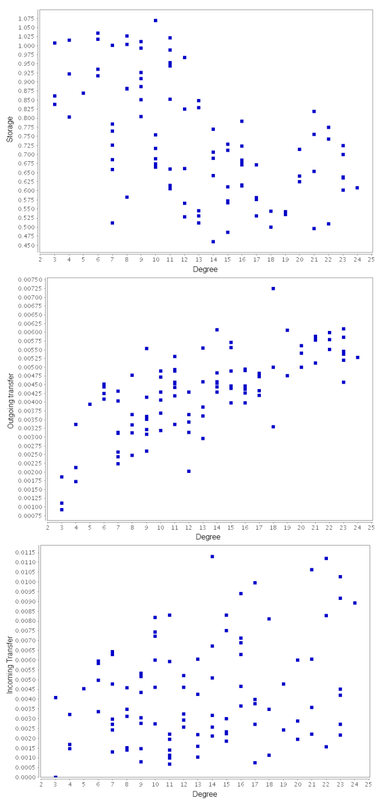
- Top: Steady decrease in storage after the critical coupling
- Bottom: Steady increase after the critical coupling
- Far past the phase transition these steady trends continue, despite hardly any change in settled state order

System Mean Dynamics vs Range



- Top: Steady decrease in storage
- Bottom: Steady increase in transfer
- Much the same as coupling, these trends continue far past the phase transition despite hardly no change in settled state order.

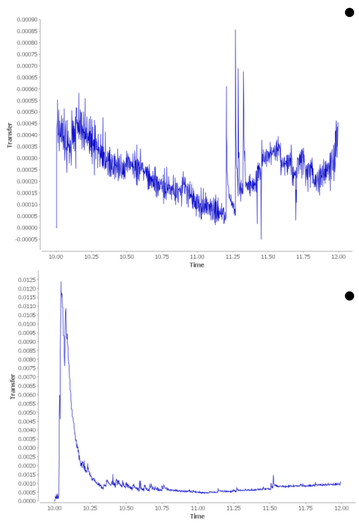
Individual Dynamics in Relation to Degree



- Top: Higher degree nodes have a correlation with nodes with low storage
- Middle: High degree nodes have a high correlation with nodes which have a high amount of average outgoing transfer
- Bottom: High degree nodes seem to have a have a higher "capacity" for average incoming transfer, but still possible to have a low amount. Compare with outgoing transfer where there are no high degree nodes with low values.

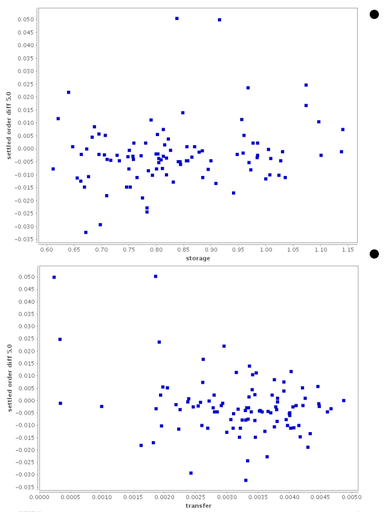
Results in Damaged Networks

Transfer over Time after Damage



- Top: Coherent system in the settled state is no different to an incoherent system, hardly any transfer is taking place.
- Bottom: After damage, information transfer spikes as the system adapts to reach a different equilibrium state

Targeted Damage effect on Settled Order



- Top: Targeting nodes with high information storage allow the system to achieve higher settled order
- Bottom: Targeting nodes with high average information transfer hinders the ability of the system to synchronise

Summary

- Interactions important to synchrony stop much earlier than existing measures show
- Despite plateauing at complete synchrony as coupling increases, the amount of information transfer continues to increase
- These measures reveal those nodes which hinder or encourage synchrony.
- These findings give us many possible interesting avenues of further exploration, eg individual dynamics over time.